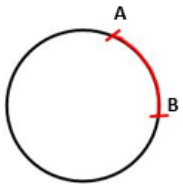
**Name: _____**

## Key Terms

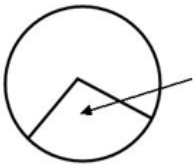
**Angle** - the space (usually measured in degrees) between two intersecting lines or surfaces at or close to the point where they meet.

**Arc** – A segment of the circumference of a circle. A **Major Arc** is the longer of the two arcs between two points on a circle. A **Minor Arc** is the shorter of the two arcs between two points on a circle.



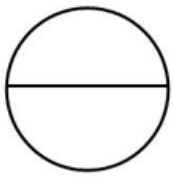

**Bisector** – A line that divides a line segment or an angle into two equal parts.

**Central Angle** – An angle whose arms are radii of a circle.

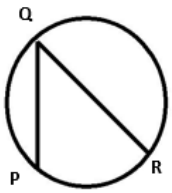


**Chord** – A line segment that joins two points on a circle.

**Diameter** - the distance across a circle, measured through its centre; or the line segment that joins two points on the circle and passes through the centre.



**Inscribed Angle** – An angle in a circle with its vertex and the endpoints of its arms on the circle.

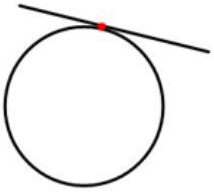



**Line Segment** – The part of a line that passes as close as possible to a set of plotted points.

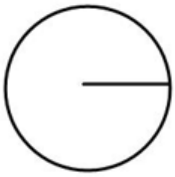
**Perpendicular** – Lines or line segments that intersect at right angles.

**Perpendicular bisector** – the line that is perpendicular to a line segment and divides it into two equal parts. A perpendicular bisector intersects a line segment at 90°.

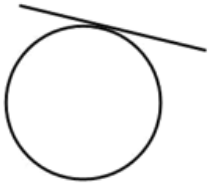
**Point of Tangency** – the point where the tangent intersects the circle.



**Radius** - the distance or line segment from the centre of a circle to any point on the circle.

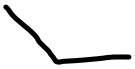


**Tangent** – a line that intersects a circle at only one point.

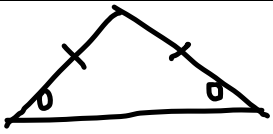


## GEOMETRY INTRODUCTION

There are five different kinds of angles. Can you name them and draw them?

| Name | Sketch | Explanation |
|---|---|---|
| Acute |  | less than 90° |
| Obtuse |  | greater than 90° but less than 180° |
| Right |  | 90° |
| Straight |  | 180° |
| Reflex |  | Greater than 180°. |

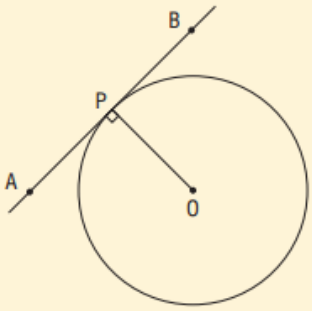
There are four different kinds of triangles (based on side length). Can you identify them?

| Name | Sketch | Explanation |
|---|---|---|
| equalateral |  | all sides and all angles are equal. |
| Right |  | one angle is 90°. |
| Scalene |  | No sides are equal |
| Isosceles |  | Two equal sides and Two equal angles. |

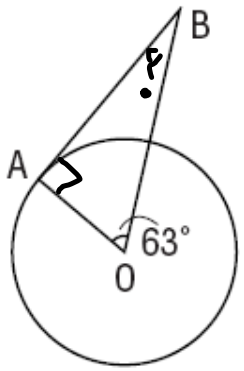
## SECTION 8.1: PROPERTIES OF TANGENTS TO A CIRCLE

**Tangent-Radius Property**

A tangent to a circle is perpendicular to the radius at the point of tangency.
That is, ∠APO = ∠BPO = 90°




**Example 1: (**Determining the measure of < ABO knowing the Tangent-Radius property)



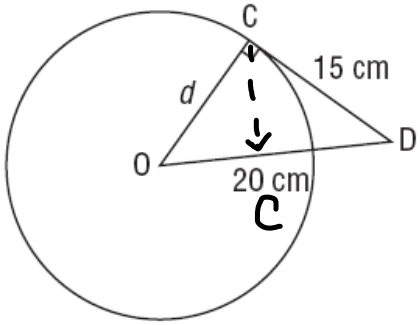

The measure of <OAB = 90°,

therefore the <ABO = 27°.

(Hint: What does the measure of all 3 angles in a triangle equal?)

180 − (90 + 63)
180 − 153
27°

**Example 2:** Show work below to solve for side d. (Determining the length of an unknown side of a right triangle knowing the Tangent-Radius property)

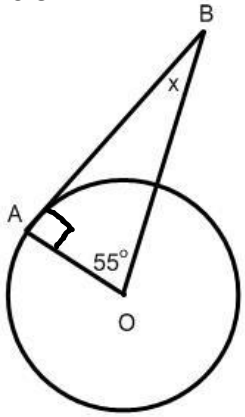



$b^2 = c^2 - a^2$
$b^2 = 20^2 - 15^2$
$b^2 = 400 - 225$
$\sqrt{b^2} = \sqrt{175}$
$b = 13.2$

∴ d = 13.2

**Practice**

1. Point O is the center of a circle and AB is tangent to the circle. In OAB, $\angle$ AOB=55$^0$. Determine the measure of $\angle$ OBA.




$\angle OBA$ :

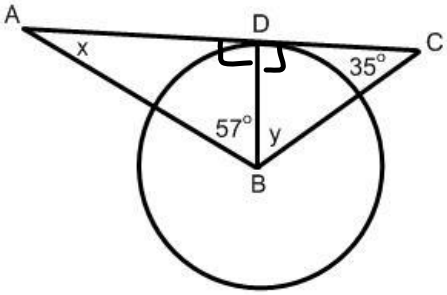
$180-(90+55)$

$=180-145$

$=35^\circ$

$\therefore \angle OBA = 35^\circ$

2. Find x and y.




x:

$180-(90+57)$

$=180-(147)$

$=33^\circ$

$\therefore \angle x = 33^\circ$

y:

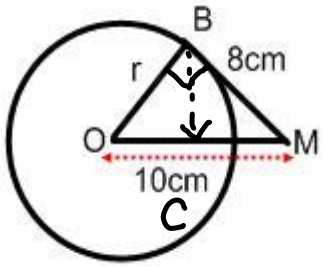
$180-(90+35)$

$=180-125$

$=55^\circ$

$\therefore \angle y = 55^\circ$

3. Find the length of side r.




$b^2=c^2-a^2$

$b^2=10^2-8^2$

$b^2=100-64$

$\sqrt{b^2}=\sqrt{36}$

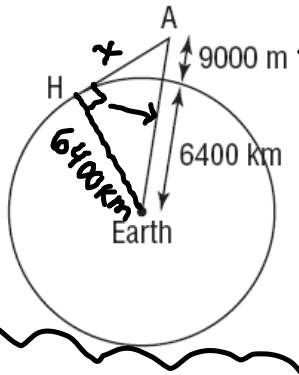
$b=6$

$\therefore r=6$

4 An airplane, A, is cruising at an altitude of 9000 m. A cross section of Earth is a circle with radius approximately 6400 km. A passenger wonders how far she is from a point H on the horizon she sees outside the window. Calculate this distance to the nearest kilometre.

KHDMDCM

9000m = 9KM




$c = 6400 + 9$
$= 6409 km$

$b^2 = c^2 - a^2$

$b^2 = 6409^2 - 6400^2$

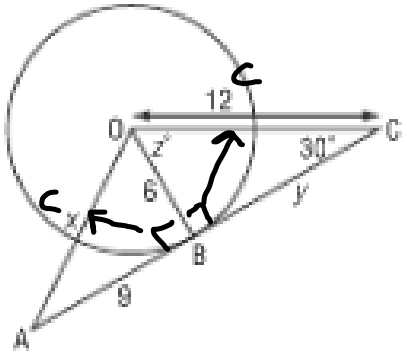
$b^2 = 41\,075\,281 - 40\,960\,000$

$\sqrt{b^2} = \sqrt{115\,281}$

$b = 339.53$

The passenger is 340 km away from point H

5. Find x, y, and z.




x:

$a^2 + b^2 = c^2$

$6^2 + 9^2 = c^2$

$36 + 81 = c^2$

$\sqrt{117} = \sqrt{c^2}$

$c = 10.8$

So, $x = 10.8$

y:

$b^2 = c^2 - a^2$

$b^2 = 12^2 - 6^2$

$b^2 = 144 - 36$

$\sqrt{b^2} = \sqrt{108}$

$b = 10.4$

So, $y = 10.4$

z:

$180 - (90 + 30)$

$= 180 - 120$

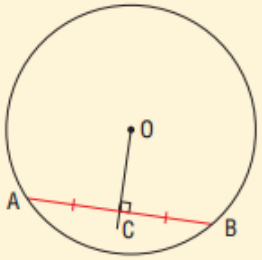
$= 60°$

$\therefore \angle Z = 60°$

## SECTION 8.2: PROPERTIES OF CHORDS IN A CIRCLE

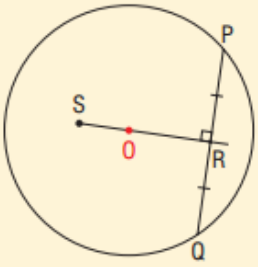
**Perpendicular to Chord Property 1**

The perpendicular from the centre of a circle to a chord bisects the chord; that is, the perpendicular divides the chord into two equal parts.
Point O is the centre of the circle.
When $\angle OCB = \angle OCA = 90°$,
then AC = CB



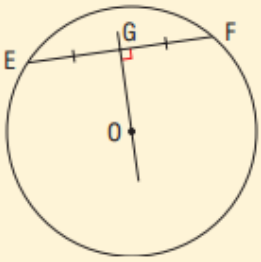

**Perpendicular to Chord Property 2**

The perpendicular bisector of a chord in a circle passes through the centre of the circle.
When $\angle SRP = \angle SRQ = 90°$ and PR = RQ,
then SR passes through O, the centre of the circle.




**Perpendicular to Chord Property 3**

A line that joins the centre of a circle and the midpoint of a chord is perpendicular to the chord.
When O is the centre of a circle and EG = GF,
then $\angle OGE = \angle OGF = 90°$

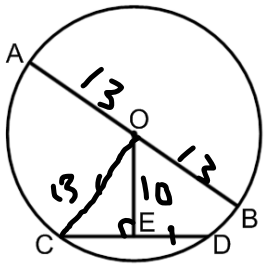



**Example 1**: O is the center of the circle. AB is the diameter and measures 26 cm. OE is 10 cm and perpendicularly bisects CD. What is the length of CD?

C
13
10

$b^2 = c^2 - a^2$
$b^2 = 13^2 - 10^2$
$b^2 = 169 - 100$
$\sqrt{b^2} = \sqrt{69}$
$b = 8.31$ ← length of CE

$d = 26$
$r = 26 \div 2 = 13$




$CD = 2 \times 8.31 = 16.6\text{ cm}$

**Example 2**: A horizontal pipe has a circular cross section, with center O. Its radius is 30cm. Water fills less than half the pipe. The surface of the water, AB, is 24 cm wide. Determine the maximum depth of the water, CD.

OC:

$b^2 = c^2 - a^2$

$b^2 = 30^2 - 12^2$

$b^2 = 900 - 144$

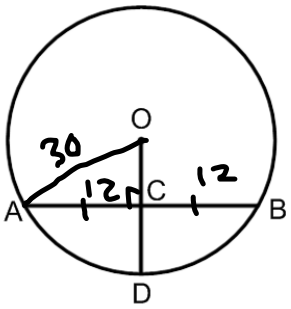
$\sqrt{b^2} = \sqrt{756}$

$b = 27.5$

CD:

$30 - 27.5$

$= 2.5\text{cm}$



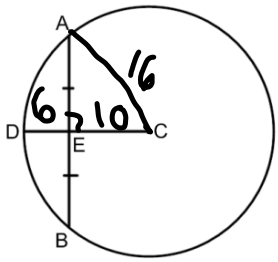

**Example 3**: The radius of the circle is 16 cm. ED is 6 cm. What is the length of AE?

$b^2 = c^2 - a^2$

$b^2 = 16^2 - 10^2$

$b^2 = 256 - 100$

$\sqrt{b^2} = \sqrt{156}$ $b = 12.5$


$AE = 12.5\text{cm}$

**Example 4**: The radius of the circle is 18 m. The radius CD is perpendicular to the chord AB. CE measures 10 m. What is the length of chord AB?

c


$b^2 = c^2 - a^2$

$b^2 = 18^2 - 10^2$

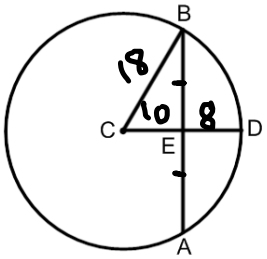
$b^2 = 324 - 100$

$\sqrt{b^2} = \sqrt{224}$

$b = 14.97$

$AB = 2 \times 14.97$

$= 30\text{m}$




HJ:

**Example 5**: Determine the value of *x*.

$b^2 = c^2 - a^2$

$b^2 = 11^2 - 5^2$

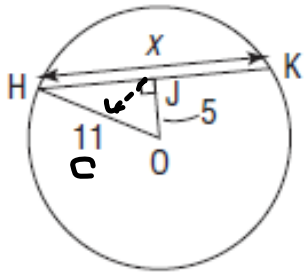
$b^2 = 121 - 25$

$\sqrt{b^2} = \sqrt{96}$

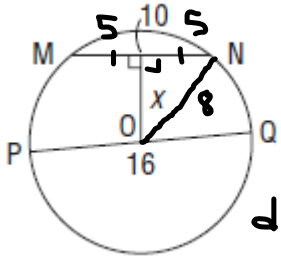
$b = 9.8$

$x = 2 \times 9.8$

$= 19.6$



**Example 6**: Determine the value of *x*.




x:

$b^2 = c^2 - a^2$

$b^2 = 8^2 - 5^2$

$b^2 = 64 - 25$

$\sqrt{b^2} = \sqrt{39}$

$b = 6.2$

So, $x = 6.2$

$d = 16$

$r = 16 \div 2 = 8$

**Example 7**: Water is flowing through a pipe with a radius of 16 cm. The maximum depth of the water is 10 cm. What is the width, PQ, at the surface of the water?

$b^2 = c^2 - a^2$

$b^2 = 16^2 - 6^2$

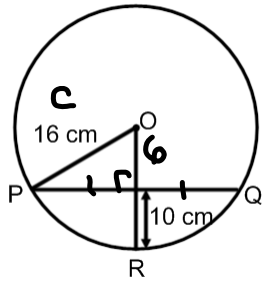
$b^2 = 256 - 36$

$\sqrt{b^2} = \sqrt{220}$

$b = 14.8$

$PQ = 2 \times 14.8$

$= 29.7\text{cm}$




**Example 8**: A square has a side length of 5 cm. It is inscribed in a circle, center. What is the length of the radius of the circle?

find the diameter:

$a^2 + b^2 = c^2$

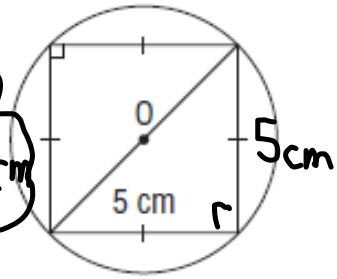
$5^2 + 5^2 = c^2$

$25 + 25 = c^2$

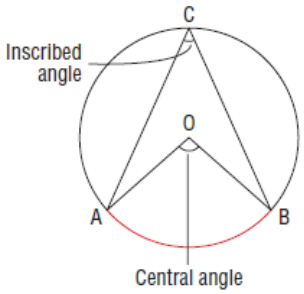
$\sqrt{50} = \sqrt{c^2}$

$c = 7.1$

So, the radius is $7.1 \div 2 = 3.5\text{cm}$



## Section 8.3: Properties of Angles in a Circle

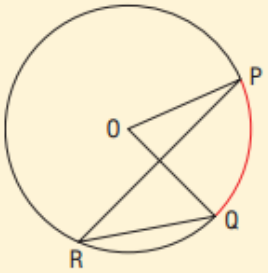



**Central Angle and Inscribed Angle Property**

In a circle, the measure of a central angle subtended by an arc is twice the measure of an inscribed angle subtended by the same arc.

$\angle POQ = 2\angle PRQ$, or

$\angle PRQ = \frac{1}{2}\angle POQ$

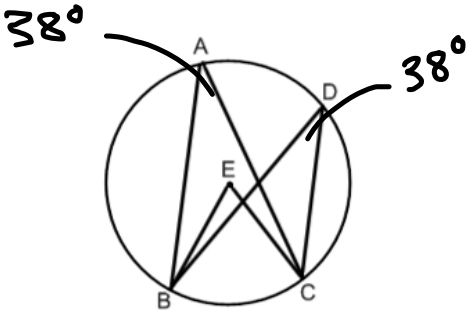



**Example 1:** $\angle$BDC is an inscribed angle and $\angle$BEC is a central angle. They are subtended by the same arc. $\angle$BDC = 38°. What is the measure of $\angle$BAC and $\angle$BEC?

$\angle BAC = 38°$

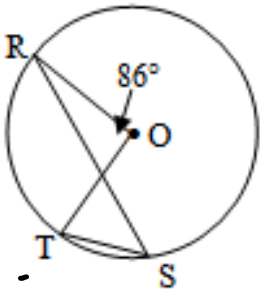
$\angle BEC = 2 \times 38$

$= 76°$



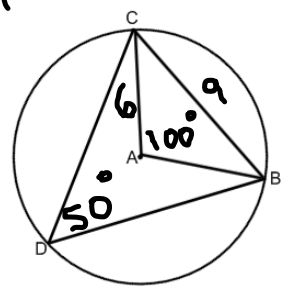

**Example 2:** $\angle$ROT is a central angle subtended by arc RT. $\angle$RST is an inscribed angle subtended by the same arc. What is the measure of $\angle$RST?

$\angle RST = 86 \div 2 = 43°$



**Example 3:** ∠CDB = 50°. CB = 9 cm and CA = 6 cm. Can you use the Pythagorean relationship to find the length of BA? Justify your answer.

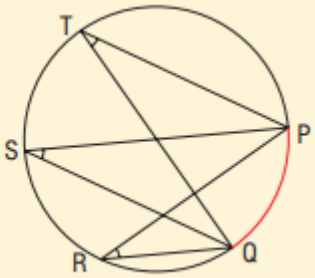
Since, $\triangle ABC$ is NOT a right triangle we cannot use Pythagorean.




**Inscribed Angles Property**

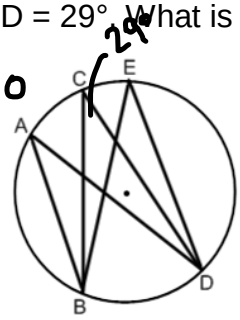
In a circle, all inscribed angles subtended by the same arc are congruent.

$\angle PTQ = \angle PSQ = \angle PRQ$



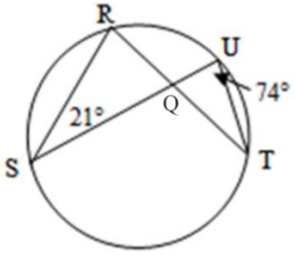

**Example 4:** ∠BCD is an inscribed angle. ∠BCD = 29°. What is the measure of ∠BAD and ∠BED?

$\angle BCD = \angle BAD = \angle BED = 29°$ they are all subtended by the same minor arc BD.




**Example 5:** ∠RSU and ∠RTU are inscribed angles subtended by the same arc. What is the measure of ∠RTU, ∠SRT and ∠UQT?

$\angle RTU = 21°$

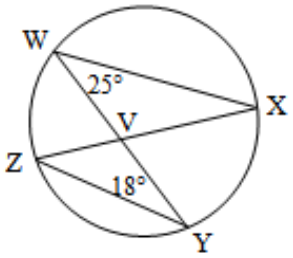



$\angle UQT = 180 - (74 + 21)$

$= 180 - 95$

$= 85°$

**Example 6:** By the inscribed angle property, the measure of ∠WXZ is 18°, and the measure of ∠XZY is 25°.

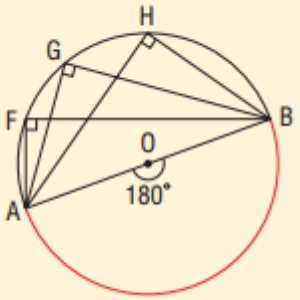



**Angles in a Semicircle Property**

All inscribed angles subtended by a semicircle are right angles.

Since ∠AOB = 180°,

then ∠AFB = ∠AGB = ∠AHB = 90°




**Example 7:** Point C is the center of the circle. The central ∠is ACD. Chord BD = 12.5 cm. The radius is 8 cm.

a) What is the measure of ∠ACD? = 180°

b) What is the measure of ∠ABD? = 90°

c) What is the length of chord AB?

d = 8 x 2 = 16

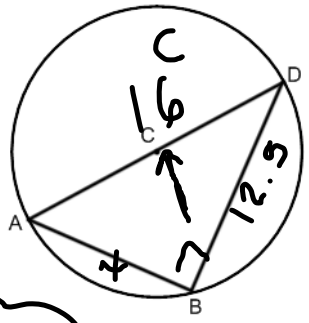
$b^2 = c^2 - a^2$

$b^2 = 16^2 - 12.5^2$

$b^2 = 256 - 156.25$

$\sqrt{b^2} = \sqrt{99.75}$

$b = 9.99$




So, chord AB = 9.99